

The $\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$ decay in Generalized χ PT

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Abstract

Calculations of $\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$ decay in Generalized chiral perturbation theory are presented. Tree level and next-to-leading corrections are involved. Sensitivity to violation of the Standard counting is discussed.

1 Introduction

The $\eta(p) \rightarrow \pi^0(p_1) \pi^0(p_2) \gamma(k) \gamma(k')$ process is a rare decay, which has been recently studied by several authors in context of Standard chiral perturbation theory (S χ PT), namely at the lowest order by Knöchlein, Scherer and Drechsel [1] and to next-to-leading by Bellucci and Isidori [2] and Ametller et al. [3]. The experimental interest for such a process comes from the anticipation of large number of η 's to be produced at various facilities.¹ The goal of our computations is to add the result for the next-to-leading order in Generalized chiral perturbation theory (G χ PT). The motivation is that one of the important contributions involve the $\eta \pi \rightarrow \eta \pi$ off-shell vertex which is very sensitive to the violation of the Standard scheme and thus this decay provides a possibility of its eventual observation. We have completed the calculations at the tree level, added 1PI one loop corrections, corrections to the $\eta \pi \rightarrow \eta \pi$ vertex and phenomenological corrections to the resonant contribution. These preliminary results we would like to present in this paper.

2 Kinematics and parameters

The amplitude of the process can be defined

$$\langle \pi^0(p_1) \pi^0(p_2) \gamma(k, \epsilon) \gamma(k', \epsilon')_{\text{out}} | \eta(p)_{\text{in}} \rangle = i(2\pi)^4 \delta^{(4)}(P_f - p) \mathcal{M}_{fi}. \quad (1)$$

In the square of the amplitude summed over the polarizations $|\overline{\mathcal{M}_{fi}}|^2 = \sum_{\text{pol.}} |\mathcal{M}_{fi}|^2$ we integrated out all of the independent Lorentz invariants except the diphoton energy square

$$s_{\gamma\gamma} = (k + k')^2, \quad 0 < s_{\gamma\gamma} \leq (M_\eta - 2M_\pi)^2. \quad (2)$$

Our goal is to calculate the partial decay width $d\Gamma$ of the η particle as the function of the diphoton energy square $s_{\gamma\gamma}$.

¹according to [4], at DAΦNE about 10^8 decays per year

At the lowest order, the S χ PT does not depend on any unknown free order parameters. In contrast, there are two free parameters controlling the violation of the Standard picture in the Generalized scheme. We have chosen them as

$$r = \frac{m_s}{\hat{m}}, \quad X_{\text{GOR}} = \frac{2B\hat{m}}{M_\pi^2} \quad (3)$$

and their ranges are $r \sim r_1 - r_2 \sim 6 - 26$, $0 \leq X_{\text{GOR}} \leq 1$. We use abbreviations for $\hat{m} = (m_u + m_d)/2$, $r_1 = 2M_K/M_\pi - 1$ and $r_2 = 2M_K^2/M_\pi^2 - 1$. The Standard values of these parameters are $r = r_2$ and $X_{\text{GOR}} = 1$.

3 Tree level

At the $O(p^4)$ tree level, the amplitude has two contributions, with a pion and an eta propagator. The first one is resonant, ‘ π^0 -pole’, the other is not, ‘ η -tail’.

The Standard values of the contributions to the partial decay rate and the maximum possible violation of the Standard counting ($r = r_1, X_{\text{GOR}} = 0$) are represented in Fig. 1. The pole of the resonant contribution at $s_{\gamma\gamma} = M_\pi^2 \sim 0.06M_\eta^2$ is transparent. While in the Standard case it is fully dominant, in the Generalized scheme the η -tail could be determining in the whole area $s_{\gamma\gamma} > 0.11M_\eta^2$. The reason can be found in the $\eta\pi \rightarrow \eta\pi$ vertex. Its contribution in the Generalized amplitude can jump up to 16 times its Standard value.

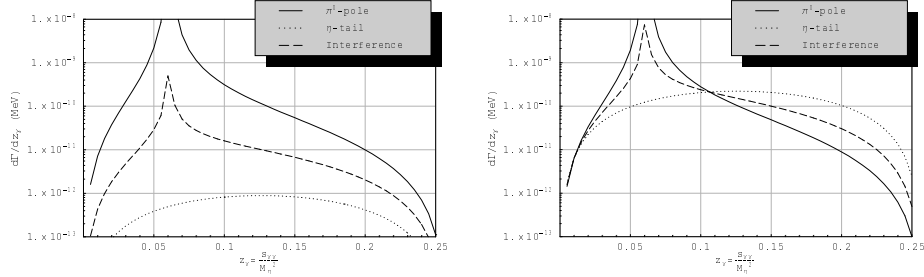


Figure 1: S χ PT and G χ PT tree level contributions to the partial decay rate $d\Gamma/dz_\gamma$

The full decay width for the Standard ($r = r_2, X_{\text{GOR}} = 1$) and Generalized case ($r = r_2, X_{\text{GOR}} = 0.5$ and $r = r_1, X_{\text{GOR}} = 0$) is displayed in Fig. 2. It can be seen, that even in the conservative intermediate case the change is quite interesting.

4 One loop corrections

There are four distinct contributions at the next-to-leading order: one loop corrections to the π^0 -pole and the η -tail, one particle irreducible diagrams (1PI) and counterterms.

In the latter case we rely upon the results of [3]. Their estimate from vector meson dominated counterterms indicates, that it causes only a slight decrease of the full decay width. Because the estimate is the same for both schemes, for our purpose of studying the differences between them we can leave it for later investigation.

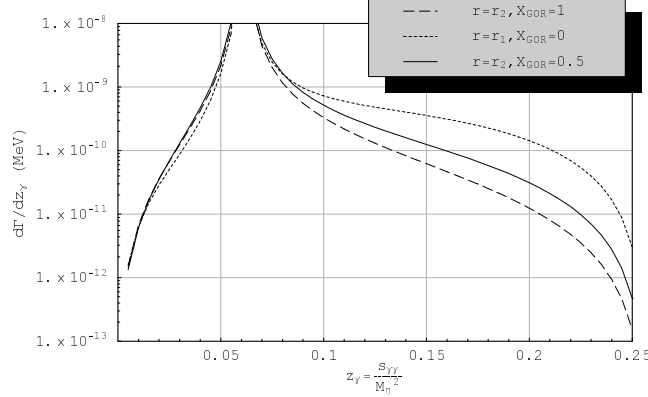


Figure 2: Full tree level decay width depending on the parameters r and X_{GOR}

More important are the corrections to the η -tail diagram. We did take into account the corrections to the $\eta\pi \rightarrow \eta\pi$ vertex. These involve loop corrections and counterterms with many unknown higher order parameters. As a first approximation, we set these parameters equal to zero and estimated their effect through the remaining dependence on the renormalization scale. The scale was moved in the range from the mass of the η to the mass of ρ -meson.

We decided, similarly to [2], to correct the π^0 -pole amplitude by a phenomenological parametrization of the $\eta \rightarrow 3\pi^0$ vertex and fix the parameters from experimental $\eta \rightarrow 3\pi^0$ data. We made an estimate of its phase by expanding the $\eta \rightarrow 3\pi^0$ one loop amplitude around the center of the Dalitz Plot. In the 1PI amplitude, we neglected the suppressed kaon loops.

Fig. 3 represents the one loop corrected decay widths for the Standard and the maximum violation of the Standard scheme. The dependence on the renormalization scale is used to estimate the uncertainty in the unknown higher order coupling constants. We can see that the scale dependence is small in the Standard counting and not too terrible in the Generalized variant. In the case of the maximum violation of $S\chi\text{PT}$, the difference is big enough to not to be washed out by the uncertainty. However, in the conservative case $r = r_2, X_{\text{GOR}} = 0.5$ this is not true and the promising results from the tree level are lost.

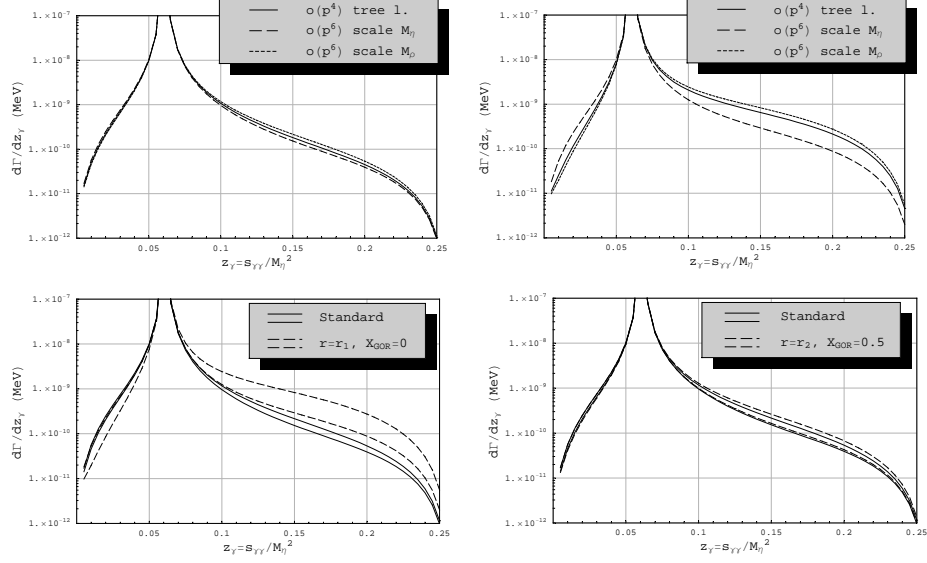


Figure 3: $S\chi PT$ and $G\chi PT$ tree level and one loop corrected full decay widths

5 Conclusion

We have analyzed the $\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$ decay to the next-to-leading order of chiral perturbation theory in its both variants. The tree level results are promising, the sensitivity to the change in parameters controlling the violation of the Standard χPT is considerable.

At the one loop level, we tried to estimate the uncertainty in the higher order couplings constants in the crucial $\eta \pi \rightarrow \eta \pi$ vertex through their dependence on the renormalization scale. Although for big violation of the Standard case the difference is preserved, for the more realistic conservative case the output is not satisfactory. We would like to stress that these results are preliminary and there are several ways how to deal with the unknown order parameters. One of them is to take into account the vector mesons, similarly to the counterterm estimate in [3]. Other way is to treat the whole χPT expansion differently, with more caution, as developed in [5]. This approach, called ‘resumed’ χPT could provide results similar to the tree level case even if the one loop corrections are involved.

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